FULLY WORKED SOLUTIONS

Chapter 7: Building it up

Chapter questions

1. $\tau = Fr \sin \theta = (5 \times 9.8) \times 0.35 \times \sin 90° = 17.15$ Nm

2. $\sin \theta = 1$

   $F = \tau / r = 150/0.3 = 5 \times 10^2$ N

3. $\tau = Fr \sin \theta$

   $5 = 30 \times 0.45 \times \sin \theta$

   $\sin \theta = 0.37$

   $\theta = 21.7°$

4. $0 = \tau_{rock} + \tau_{push}$

   $0 = -(20 \times 9.8) \times 0.5 + F \times 2.5$

   $0 = -98 + 2.5F$

   $98 = 2.5F$

   $F = 39.2$ N

5. $w_{Tasmania} = 784$ N

   Let R_1 be the reaction force at the end closest to Tasmania and R_2 the reaction force at the end furthest from Tasmania.

   Considering rotational equilibrium around R_2:

   $\tau_{R2} = 0$

   $0 = 4R_1 - (2.5 \times 784) - (2 \times 2000)$

   $0 = 4R_1 - 5960$

   $5960 = 4R_1$

   $R_1 = 1490$ N

6. (a) $w_{plank} = 40 \times 9.8 = 392$ N

   $w_{painter} = 50 \times 9.8 = 490$ N

   Considering translational equilibrium:

   $\Sigma F = 0$

   $R_1 + R_2 = w_{plank} + w_{painter} = 490 + 392$

   $R_1 + R_2 = 882 \text{ N} \qquad (1)$

   Considering rotational equilibrium around trestle 2:

   $\Sigma \tau_2 = 0$

   $0 = 3R_1 - (2 \times 490) - (1.5 \times 392)$

$0 = 3R_1 - 1568$

$1568 = 3R_1$

$R_1 = 523$ N (2)

(b) Substituting (2) into (1):

$523 + R_2 = 882$

$R_2 = 359$ N

7. Considering translational equilibrium:

$\Sigma F = 0$

$0 = R_A + R_B - 1000$

$R_A + R_B = 1000$ (1)

Considering rotational equilibrium around point B:

$\Sigma \tau_B = 0$

$0 = 4R_A - (3 \times 1000)$

$3000 = 4R_A$

$R_A = 750$ N (2)

Substituting (2) into (1):

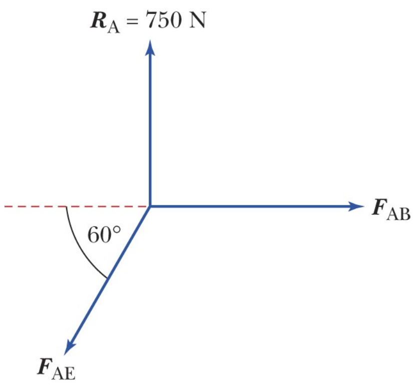
$750 + R_B = 1000$

$R_B = 250$ N

Consider the forces acting on point A:

$\Sigma F_A = 0$

To start, assume that F_{AE} is compression force and F_{AB} is a tension force.



Resolving in the x-direction:

$0 = 750 \cos 90° + F_{AE} \cos 240° + F_{AB} \cos 0°$

$0 = 0 - 0.5F_{AE} + F_{AB}$

$F_{AB} = 0.5F_{AE}$

Resolving in the y-direction:

$0 = 750 \sin 90° + F_{AE} \sin 240° + F_{AB} \sin 0°$

$0 = 750 - 0.866\, F_{AE}$

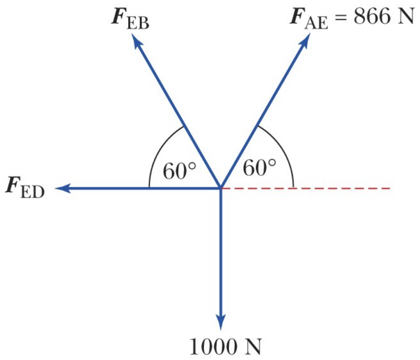
$F_{AE} = 866$ N (compression)

$F_{AB} = 0.5F_{AE} = 0.5 \times 866 = 433$ N (tension)

Consider the forces acting on point E:

$\Sigma F_E = 0$

Assume that F_{EB} and F_{ED} are compression forces.



Resolving in the x-direction:

$0 = 866 \cos 60° + F_{EB} \cos 120° + F_{ED} \cos 180° + 1000 \cos 270°$

$0 = 433 - 0.5\, F_{EB} - F_{ED}$

$F_{ED} = 433 - 0.5\, F_{EB}$

Resolving in the y direction:

$0 = 866 \sin 60° + F_{EB} \sin 120° + F_{ED} \sin 180° + 1000 \sin 270°$

$0 = 750 + 0.866F_{EB} + 0 - 1000$

$250 = 0.866F_{EB}$

$F_{EB} = 288.7$ N (compression)

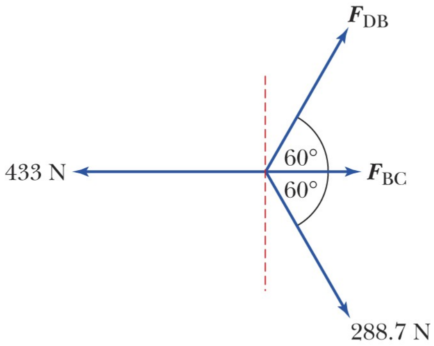
$F_{ED} = 433 - 0.5F_{EB} = 433 - (0.5 \times 288.7)$

$F_{ED} = 288.7$ N (compression)

Consider the forces acting on point B:

$\Sigma F_B = 0$

Assume F_{DB} and F_{BC} are tension forces.



Resolving in the x-direction:

$0 = F_{BC} \cos 0° + F_{DB} \cos 60° + 433 \cos 180° + 288.7 \cos 300°$

$0 = F_{BC} + 0.5F_{DB} - 433 + 144.3$

$F_{BC} = 288.7 - 0.5F_{DB}$

Resolving in the y-direction:

$0 = F_{BC} \sin 0° + F_{DB} \sin 60° + 433\sin 180° + 288.7 \sin 300°$

$0 = 0 + 0.866F_{DB} + 0 - 250$

$250 = 0.866F_{DB}$

$F_{DB} = 288.7$ N (tension)

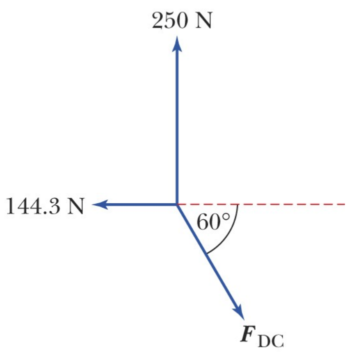
$F_{BC} = 288.7 - 0.5F_{DB} = 288.7 - (0.5 \times 288.7)$

$F_{BC} = 144.3$ N (tension)

Finally, consider the forces acting on point C:

$\Sigma F_C = 0$

Assume F_{DC} is a compression force.



Resolving in the x-direction:

$0 = 250 \cos 90° + 144.3 \cos 180° + F_{DC} \cos 300°$

$0 = 0 - 144.3 + 0.5F_{DC}$

$144.3 = 0.5F_{DC}$

$F_{DC} = 288.7$ N (compression)

Review questions

12. Let the support on the left be A and the one on the right B.

$\Sigma \boldsymbol{F} = 0$

$\boldsymbol{R}_A + \boldsymbol{R}_B = 800 + 600 + 200 + 2000$

$\boldsymbol{R}_A + \boldsymbol{R}_B = 3600$ N

$\Sigma \boldsymbol{\tau}_A = 0$

$0 = (800 \times 2) + (600 \times 4) + (200 \times 5) + (2000 \times 3) - (\boldsymbol{R}_B \times 6)$

$6\boldsymbol{R}_B = 11\ 000$

$\boldsymbol{R}_B = 1\ 833$ N

$\boldsymbol{R}_A = 3600 - 1833 = 1767$ N

13. (a) 4 mm

(b) 16 mm

(c) 54 mm

15. $\mathbf{w}_g = 50 \times 9.8 = 490$ N

$\mathbf{w}_b = 250 \times 9.8 = 2\ 450$ N

As $\Sigma \boldsymbol{F} = 0$,

$\boldsymbol{R}_1 + \boldsymbol{R}_2 = w_g + \boldsymbol{w}_b$

$\boldsymbol{R}_1 + \boldsymbol{R}_2 = 490 + 2450 = 2940$ N

$\Sigma \boldsymbol{\tau}_1 = 0$

$0 = (2450 \times 1.96) + (490 \times 4.46) - (\boldsymbol{R}_2 \times 3.92)$

$3.92\boldsymbol{R}_2 = 6987.4$

$\boldsymbol{R}_2 = 1\ 782$ N

$\boldsymbol{R}_1 = 2940 - 1782 = 1\ 158$ N

16. $\boldsymbol{\tau} = 10$ N m, $r = 400$ mm $= 0.4$ m

$$\boldsymbol{F} = \frac{\tau}{r} = \frac{10}{0.4} = 25 \text{ N}$$

17. $\boldsymbol{F}_{\text{nut}} \times \boldsymbol{r}_{\text{nut}} = \boldsymbol{F}_{\text{end}} \times \boldsymbol{r}_{\text{end}}$

$40 \times 0.026 = \boldsymbol{F}_{\text{end}} \times 0.12$

$\boldsymbol{F}_{\text{end}} = 8.7$ N

18. $\Sigma \boldsymbol{F}_{\text{jeans}} = 0$

$\boldsymbol{w}_{\text{jeans}} = 0.9 \times 9.8 = 8.82$ N

Considering the vertical components of the forces acting on the jeans:

$0 = 8.82 \sin 270° + \boldsymbol{T} \sin 8° + \boldsymbol{T} \sin 172°$

$0 = -8.82 + 0.14\boldsymbol{T} + 0.14\boldsymbol{T}$

$8.82 = 0.28\boldsymbol{T}$

$\boldsymbol{T} = 31.5$ N

19. As there is no movement at the hinge, $\Sigma\boldsymbol{\tau}_{\text{hinge}} = 0$

$0 = (400 \times 12 \times \sin 30°) - (\boldsymbol{w} \times 6 \times \sin 60°)$

$0 = 2400 - 5.2\boldsymbol{w}$

$5.2\boldsymbol{w} = 2\,400$

$\boldsymbol{w} = 462$ N

20. $m_{\text{Sean}} = 70$ kg; $m_{\text{bridge}} = 2.7$ kN $= 2\,700$ N

$\boldsymbol{w}_{\text{Sean}} = 70 \times 9.8 = 686$ N

As $\Sigma\boldsymbol{F} = 0$

$\boldsymbol{R}_{\text{near}} + \boldsymbol{R}_{\text{far}} = \boldsymbol{w}_{\text{Sean}} + \boldsymbol{w}_{\text{bridge}}$

$\boldsymbol{R}_{\text{near}} + \boldsymbol{R}_{\text{far}} = 686 + 2\,700$

$\boldsymbol{R}_{\text{near}} + \boldsymbol{R}_{\text{far}} = 3386$ N

$\Sigma\boldsymbol{\tau}_{\text{near}} = 0$

$0 = -(686 \times \frac{L}{4}) - (2700 \times \frac{L}{2}) + (\boldsymbol{R}_{\text{far}} \times L)$

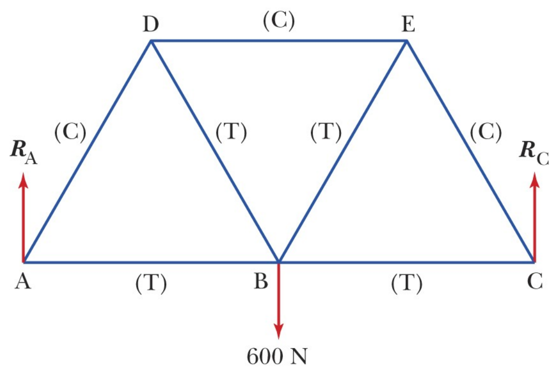
$0 = -171.5L - 1350L + \boldsymbol{R}_{\text{far}}L$

$0 = -1521.5L + \boldsymbol{R}_{\text{far}}L$

$1521.5L = \boldsymbol{R}_{\text{far}}L$

Therefore, $\boldsymbol{R}_{\text{far}} = 1521.5$ N

21. Let the truss joints be labelled as shown here, and the type of force we initially assume each strut to experience marked as C or T.



As the entire truss is in equilibrium, $\Sigma\boldsymbol{F} = 0$

$\boldsymbol{R}_A + \boldsymbol{R}_C = 600$ N

The symmetrical nature of the truss and the location of the load, indicate that $\boldsymbol{R}_A$ and $\boldsymbol{R}_C$ are equal in size, therefore

$\boldsymbol{R}_A = 300$ N and $\boldsymbol{R}_C = 300$ N

The forces at each joint are also in equilibrium. Let's consider the forces at joint A.

$\Sigma \boldsymbol{F}_A = 0$

Resolving forces horizontally,

$0 = \boldsymbol{R}_A \cos 90° + \boldsymbol{F}_{AD} \cos 240° + \boldsymbol{F}_{AB} \cos 0°$

$0 = 0 - 0.5\, \boldsymbol{F}_{AD} + \boldsymbol{F}_{AB}$

$0.5\, \boldsymbol{F}_{AD} = \boldsymbol{F}_{AB}$ (1)

Resolving forces vertically:

$0 = \boldsymbol{R}_A \sin 90° + \boldsymbol{F}_{AD} \sin 240° + \boldsymbol{F}_{AB} \sin 0°$

$0 = 300 - 0.866\, \boldsymbol{F}_{AD} + 0$

$0.866\, \boldsymbol{F}_{AD} = 300$

$\boldsymbol{F}_{AD} = 346.4$ N

By substitution into (1), we get

$0.5 \times 346.4 = \boldsymbol{F}_{AB}$

$\boldsymbol{F}_{AB} = 173.2$ N

Note by a similar process we will find that, symmetrically, $\boldsymbol{F}_{CE} = 346.4$ N and $\boldsymbol{F}_{CB} = 173.2$ N.

Now let's look at joint B.

Resolving forces horizontally:

$0 = \boldsymbol{F}_{CB} \cos 0° + \boldsymbol{F}_{BE} \cos 60° + \boldsymbol{F}_{BD} \cos 120° + \boldsymbol{F}_{AB} \cos 180° + 600 \cos 270°$

$0 = 173.2 + 0.5\, \boldsymbol{F}_{BE} - 0.5\, \boldsymbol{F}_{BD} - 173.2 + 0$

$\boldsymbol{F}_{BE} = \boldsymbol{F}_{BD}$

Resolving forces vertically:

$0 = \boldsymbol{F}_{CB} \sin 0° + \boldsymbol{F}_{BE} \sin 60° + \boldsymbol{F}_{BD} \sin 120° + \boldsymbol{F}_{AB} \sin 180° + 600 \sin 270°$

$0 = 0 + 0.866\, \boldsymbol{F}_{BE} + 0.866\, \boldsymbol{F}_{BD} + 0 - 600$

$600 = 0.866\, \boldsymbol{F}_{BE} + 0.866\, \boldsymbol{F}_{BD}$

As we know that $\boldsymbol{F}_{BE} = \boldsymbol{F}_{BD}$

$600 = 0.866\, \boldsymbol{F}_{BE} + 0.866\, \boldsymbol{F}_{BE}$

$600 = 1.732\, \boldsymbol{F}_{BE}$

$\boldsymbol{F}_{BE} = 346.4$ N

Also, $\boldsymbol{F}_{BD}$ = 346.4 N

Finally, considering joint D:

Resolving forces horizontally:

$0 = \boldsymbol{F}_{DE} \cos 180° + \boldsymbol{F}_{AD} \cos 60° + \boldsymbol{F}_{BD} \cos 300°$

$0 = - \boldsymbol{F}_{DE} + 346.4 \cos 60° + 346.4 \cos 300°$

$0 = - \boldsymbol{F}_{DE} + 173.2 + 173.2$

$\boldsymbol{F}_{DE} = 346.4$ N

22. (a) m = 10 t = 10 000 kg

$\boldsymbol{F} = \boldsymbol{w} = m\boldsymbol{g} = 10\,000 \times 9.8 = 98\,000$ N

At the centre point, $\Sigma \boldsymbol{F} = 0$

Resolving the forces vertically;

$0 = 98\,000 \sin 270° + \boldsymbol{T} \sin 30° + \boldsymbol{T} \sin 150°$

$0 = - 98\,000 + 0.5\ \mathrm{T} + 0.5\boldsymbol{T}$

$\boldsymbol{T} = 98\,000$ N

(b) Considering the point at the top of one of the pylons, and resolving vertically:

$0 = 98\,000 \sin 330° + \boldsymbol{P} \sin 90° + 98\,000 \sin 210°$

$0 = - 49\,000 + \boldsymbol{P} – 49\,000$

$\boldsymbol{P} = 98\,000$ N

23. The centre of mass was originally located 0.455 m from both the hinge and the catch. It has been moved so that the distance from the hinge is now 0.355 m, making it 0.555 m from the catch.

$\mathbf{w} = m\boldsymbol{g} = 11 \times 9.8 = 107.8$ N

(a) $\Sigma \boldsymbol{\tau}_{catch} = 0$

$0 = –(107.8 \times 0.555) + (\boldsymbol{F}_{hinge} \times 0.91)$

$0 = –59.8 + 0.91\boldsymbol{F}_{hinge}$

$59.8 = 0.91\boldsymbol{F}_{hinge}$

$\boldsymbol{F}_{hinge} = 65.8$ N

(b) $\Sigma \boldsymbol{\tau}_{hinge} = 0$

$0 = (107.8 \times 0.355) – (\boldsymbol{F}_{catrch} \times 0.91)$

$0 = 38.3 – 0.91\ \boldsymbol{F}_{catch}$

$\boldsymbol{F}_{catch} = 42$ N

24. (a) Cross-section of wall, $A = 0.2\ \mathrm{m} \times 2\ \mathrm{m} = 0.4\ \mathrm{m}^2$

Cross-section of footing, $A = 0.6 \text{ m} \times 0.4 \text{ m} = 0.24 \text{ m}^2$

Total A of footing and wall = $0.4 + 0.24 = 0.64 \text{ m}^2$

The centre of mass will be located midway through the wall and at a height whereby an equal cross-sectional area will lie above and below it. Thus it will be at a height where there will be a cross-section of 0.32 m^2 above and below it.

Let h be the height of the centre of mass above the footing.

Then, $0.32 = 0.24 + (h \times 0.2)$

$0.08 = 0.2h$

$h = 0.4 \text{ m}$

Thus, the centre of mass will be located 400 mm above the top of the footing, or 1600 mm down from the top of the wall.

(c) The wind will strike at a point 1.0 m from the top of the footing, making it 1.4 m from the bottom of the footing.

At the pivot point, $\Sigma\boldsymbol{\tau} = 0$.

$0 = (\boldsymbol{w}_{\text{wall}} \times 0.5d_{\text{footing}}) - (\boldsymbol{F}_{\text{wind}} \times h)$

$0 = (200\,000 \times 0.3) - (\boldsymbol{F}_{\text{wind}} \times 1.4)$

$1.4\ \boldsymbol{F}_{\text{wind}} = 60\,000$

$\boldsymbol{F}_{\text{wind}} = 42\,860 \text{ N}$